Y12 – Chapter 1 Algebraic Expressions

What do I need to be able to do?

By the end of this chapter you should be able to:

- Multiply and divide integer powers
- Expand a single term of brackets and collect like terms
- Expand the product of two or three expressions
- Factorise linear, quadratic and simple cubic expressions
- Know and use the laws of indices
- Simplify and use the rules of surds
- Rationalise denominators

Key words:

- Integer – A number with no fractional part (no decimals)
- Product – The answer when two or more values are multiplied together
- Surd – A number that can't be simplified to remove a square root (or cube root etc)
- Irrational – A real number that can NOT be made by dividing two integers eg π
- Rational – A number that can be made by dividing two integers
- Base – The number that gets multiplied when using an exponent. (index/power)

Expanding and factorising

Expanding and factorising are the inverse of each other

Expanding brackets →

$4x(2x+y) = 8x^2 + 4xy$

$(x+5)^3 = x^3 + 15x^2 + 75x + 125$

$(x+2y)(x-5y) = x^2 - 3xy - 10y^2$

← Factorising

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Surds

Writing surds in their simplest form

If a square root has a perfect square number as a factor, then it can be simplified.

e.g. $\sqrt{20}$ can be re-written as $\sqrt{4}\times\sqrt{5}$ which simplifies to $2\sqrt{5}$

Perfect square ($\sqrt{4}$)

Adding and subtracting surds

Remember to add or subtract like terms (i.e. the rational numbers and the roots (of the same number))

e.g. $(7+3\sqrt{2})+(8-\sqrt{2})=15+2\sqrt{2}$ Add rational parts: $(7+8=15)$

Add roots: $(3\sqrt{2}-1\sqrt{2}=2\sqrt{2})$

Multiplying surds

If there is no rational part then multiplying is easy: e.g. $\sqrt{3}\times\sqrt{5}=\sqrt{15}$

If there is a rational part then multiply out the brackets

e.g.

Remember that $\sqrt{3}\times\sqrt{3}=3$

$(5+\sqrt{3})\times(2-\sqrt{3})=10-5\sqrt{3}+2\sqrt{3}-\sqrt{3}\sqrt{3}$ tidies up to give $7-3\sqrt{3}$

Rationalising the denominator

You rationalise the denominator to get rid of the surd on the bottom of a fraction.

To rationalise the denominator just multiply the top and bottom of the fraction by the bottom of the fraction with the opposite sign in front of the root.

e.g. $\dfrac{3+\sqrt{5}}{2-\sqrt{5}}$

We are just finding an equivalent fraction by multiplying by 1 (just in disguise!)

$$\frac{3+\sqrt{5}}{2-\sqrt{5}}\times\frac{2+\sqrt{5}}{2+\sqrt{5}} = \frac{6+3\sqrt{5}+2\sqrt{5}+\sqrt{5}\sqrt{5}}{4+2\sqrt{5}-2\sqrt{5}-\sqrt{5}\sqrt{5}} = \frac{11+5\sqrt{5}}{-1} = -11-5\sqrt{5}$$

Notice these are the same – but the sign in front of the root has changed

Changing the sign in front of the root makes the middle parts cancel each other out

Indices

An index (power) tells you how many times to multiply something by itself:

e.g. x^5 means $x \times x \times x \times x \times x$

There is a base and a power eg:

base → a^m ← power

Rule	Meaning
$a^m \times a^n = a^{m+n}$	To multiply 2 numbers with the same base you add the powers.
$\frac{a^m}{a^n} = a^{m-n}$	To divide 2 numbers with the same base you subtract the powers.
$(a^m)^n = a^{mn}$	To simplify a power inside and outside of a bracket you multiply the powers.
$a^{-m} = \frac{1}{a^m}$	A negative power means find the reciprocal ("one over") so send everything to the bottom of a fraction.
$a^{\frac{m}{n}} = (\sqrt[n]{a})^m$	A fractional power means a root. Denominator tells you the root and the numerator tells you the power.
$a^0 = 1$	Anything to the power of zero = 1
$a^1 = a$	Any number to the power of one stays the same

Y12 – Chapter 2 Quadratics

What do I need to be able to do?

By the end of this chapter you should be able to:

- Solve quadratic equations using factorisation, the quadratic formula and completing the square
- Read and use f(x) notation when working with functions
- Sketch the graph and find the turning point of a quadratic function
- Find and interpret the discriminant of a quadratic expression
- Use and apply models that involve quadratic functions

Solving quadratic equations

Remember that to solve a quadratic equation you should collect all the terms on one side so that the other side of the equation is 0.
When you solve the equation, it you have found the roots (ie. where the graph of the quadratic function crosses the $\boldsymbol{x}$-axis).

Factorising
Put the quadratic into brackets. If the product of two expressions is zero one or both of them must be equal to zero.
E.g. Solve $x^2+6x+8=0$

$$(x+4)(x+2)=0$$

We need two numbers that add to make the coefficient of x and multiply to give the constant term

$x+4=0$ or $x+2=0$

Therefore: $x=-4$ or $x=-2$

The quadratic formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

E.g. Solve $3x^2-7x-1=0$

a = 3 b = -7 c = -1

Substitute into the formula:

$$x=\frac{-(-7)\pm\sqrt{(-7)^2-4\times(3)\times(-1)}}{2\times(3)}$$

Put each number in a bracket to avoid any sign errors

Therefore: $x=\frac{7+\sqrt{61}}{6}$ or $x=\frac{7-\sqrt{61}}{6}$

Make sure you give your answer in the form asked for. If they want exact leave in surd for like this. If they say 3sf or 1dp then make sure you give the decimal form of the answer

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Key words:

- Quadratic – Where the highest exponent (index/power) of the variable is a square (2)
- Function – A special relationship where each input has a single output. It is often written as "f(x)" where x is the input value
- Domain – All the values that go into a function
- Range – The set of all output values of a function
- Discriminant – The expression b2 – 4ac used when solving Quadratic Equations. It can "discriminate" between the possible types of answer

The general shape of a quadratic graph:

$y=x^2$  $y=-x^2$ 

Completing the square

Completing the square can be used to solve a quadratic equation but it is also very useful in determining the turning point of a quadratic function

The completed square form looks like this:

$$A(x+B)^2+C=0$$

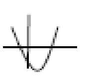
Where the turning point is (-B, C)

Remember! If you need to solve the quadratic to find the roots and it is already in the completed square form, you don't need to factorise or use the formula you can just rearrange to find $\boldsymbol{x}$.

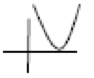
The discriminant

The expression inside the square root sign is called the discriminant and tells you what type of roots to expect.

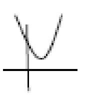
If $\boldsymbol{b^2-4ac>0}$ there are 2 real roots (ie. the curve crosses the $\boldsymbol{x}$-axis in 2 places)



If $\boldsymbol{b^2-4ac=0}$ there is 1 real root (ie. the curve touches the $\boldsymbol{x}$-axis in 1 place)



If $\boldsymbol{b^2-4ac<0}$ there are no real roots (ie. the curve does not cross the $\boldsymbol{x}$-axis)



What do I need to be able to do?

By the end of this chapter you should be able to:

- Solve linear simultaneous equations using elimination or substitution
- Solve simultaneous equations: one linear and one quadratic
- Interpret algebraic solutions of equations graphically
- Solve linear and quadratic inequalities
- Interpret inequalities graphically
- Represent linear and quadratic inequalities graphically

Y12 – Chapter 3 Equations and inequalities

Key words:

- Simultaneous equations – Two or more equations that share variables
- Equation – a mathematical statement containing an equals sign, to show that two expressions are equal. An equation will have a finite set of solutions
- Inequality – An inequality compares two values, showing if one is less than, greater than, or simply not equal to another value

Solving simultaneous equations

Method	Explanation	Works for
Elimination	Make the coefficients of one of the unknowns the same. (whichever seems easier) Add or subtract the equations to eliminate one unknown Solve the new equation to find the first unknown. Substitute back into one of the original equations to find the other unknown.	Linear simultaneous equations
Substitution	Rearrange one of the equations (if necessary) to make either x or y the subject. Substitute into the other equation. Solve the new equation to find x or y. Substitute back into your rearranged equation to find the value of the other letter. *If after substituting you get a quadratic equation you can use the discriminant to determine the number of solutions	Linear only and one linear and one quadratic simultaneous equations
Graphically	On the same set of axes draw the graphs of both simultaneous equations The points of intersection will give you the solutions	Linear only and one linear and one quadratic simultaneous equations

Linear inequalities

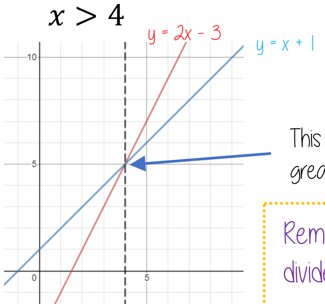
We solve linear inequalities the same way we would solve equations, except you get a range of solutions instead of one particular solution.

Eg Solve the inequality $2x - 3 > x + 1$ and sketch the outcome on a graph.

$2x - 3 > x + 1$

$2x > x + 4$

$x > 4$



This is the point where 2x-3 becomes greater than x + 1

Remember! If you multiply or divide an inequality by a negative number you have to reverse the inequality sign

Quadratic inequalities

To solve a quadratic inequality: <u>always</u> do a quick sketch (you will need to know the shape and the roots) then look for the appropriate part of the graph (ie. <0 (below the x-axis) or >0 (above the x-axis) depending on what you are looking for).

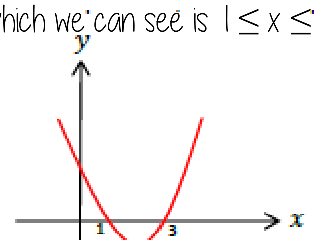
Eg Solve the inequality $x^2 + 4x + 3 \leq 0$

$x^2 + 4x + 3 = 0$

$(x + 3)(x + 1) = 0$

$x = -3$ or $x = -1$ ← These are the roots

We want the graph to be ≤ 0 so we want to describe the x values that represent the part of the curve under the x axis which we can see is $1 \leq x \leq 3$



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Y12 – Chapter 4 Graphs and transformations

Key words:

- Cubic function– A function where the highest exponent (index/power) of the variable is a cube (3)
- Quartic function – A function where the highest exponent (index/power) of the variable is 4
- Reciprocal function – A function where the highest exponent (index/power) of the variable is negative
- Asymptote – A line that a curve approaches, as it heads towards infinity.

What do I need to be able to do?

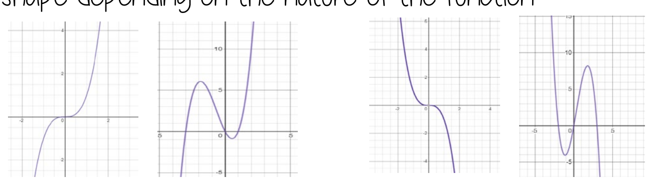
By the end of this chapter you should be able to:

- Sketch cubic, quartic and reciprocal graphs
- Use intersection points to solve equations
- Translate graphs
- Stretch graphs
- Transform graphs of unfamiliar functions

Cubic graphs

Have the form: $ax^3 + bx^2 + cx + d$ where a, b, c and d are real numbers and a is non-zero

A cubic graph can have varying forms of the same basic shape depending on the nature of the function



For these two function a is positive   For these two function a is negative

Finding the roots and y intercept of the function helps sketch the function.

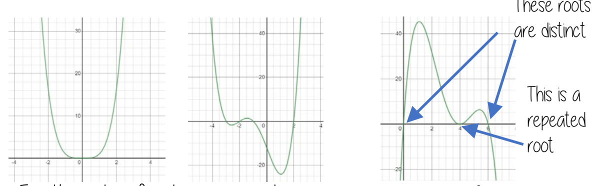
To find the roots substitute y = 0 into the function and solve

To find the y intercept substitute x=0 into the function and solve

Quartic graphs

Have the form: $ax^4 + bx^3 + cx^2 + dx + e$ where a, b, c, d and e are real numbers and a is non-zero

A quartic graph can have varying forms of the same basic shape depending on the nature of the function



For these two function a is positive   For these two function a is negative

Finding the roots and y intercept of the function helps sketch the function.

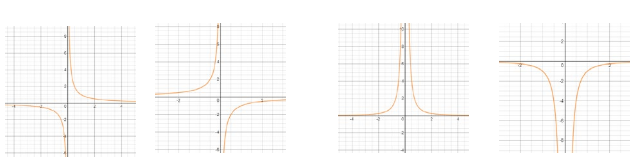
To find the roots substitute y = 0 into the function and solve

To find the y intercept substitute x=0 into the function and solve

Reciprocal graphs

Have the form: $\frac{k}{x}$ or $\frac{k}{x^2}$ where k is a real constant.

Reciprocal graphs will have asymptotes. Reciprocal graphs in the form $\frac{k}{x}$ or $\frac{k}{x^2}$ will have asymptotes as x=0 and y =0



$y = \frac{k}{x}$ with k>0 $y = \frac{k}{x}$ with k<0 $y = \frac{k}{x^2}$ with k>0 $y = \frac{k}{x^2}$ with k<0

Transformations of functions

Function	Transformation	Explanation
f(x)	None - original function	n/a
f(x) + a	Translation	Graph moves along y axis by the vector $\binom{0}{a}$
f(x+a)	Translation	Graph moves along x axis by the vector $\binom{-a}{0}$
af(x)	Stretch	Scale factor a in the vertical direction
f(ax)	Stretch	Scale factor $\frac{1}{a}$ in the horizontal direction
-f(x)	Reflection	Reflection of f(x) in the x-axis
f(-x)	Reflection	Reflection of f(x) in the y-axis

Y12 – Chapter 5 Straight line graphs

What do I need to be able to do?

By the end of this chapter you should be able to:

- Calculate the gradient of a line
- Understand the link between the equation of a line and its gradient and y-intercept
- Find the equation of a line
- Find the points of intersection of straight lines
- Know and use the rules for parallel and perpendicular gradients
- Solve length and area problems
- Use straight line graphs to construct mathematical models

Key words:

- Gradient – How steep a line is
- Y-intercept – The point where a line or curve crosses the y-axis of a graph
- Parallel – Always the same distance apart and never touching
- Perpendicular – At right angles (90°) to
- Linear equation – An equation that makes a straight line when it is graphed

Parallel or perpendicular?

Parallel lines – have the same gradient

Perpendicular lines – the product of the gradients is -1 (the gradients are negative reciprocals of each other)

The equation of a straight line

There are several ways you can write an equation of a straight line:

Form	Why it's useful
$y=mx+c$	The most commonly used form where m is the gradient and c the y-intercept
$y-y_1=m(x-x_1)$	When you have the gradient and a single point on the line, substitute them in for m, y_1 and x_1 - rearrange if necessary
$ax+by+c=0$	Useful when the gradient is a fraction and you want integer values

Finding the distance between two point

Find the distance between (x_1, y_1) and (x_2, y_2) - Pythagoras' theorem

$$\text{Distance} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Sketching a straight line

If you are given two points on the line, plot them and draw a line going through them

If you are given the equation in the form y=mx+c plot the y intercept and then use the gradient to find additional points and join up

If you are given the equation in the form ax+by+c=0, find the x intercept (sub in y=0) and the y intercept (x=0), plot and join

Finding the gradient of a straight line

The gradient (m) of the line that joins the points (x_1, y_1) and (x_2, y_2) use the formula:

$$m=\frac{y_2-y_1}{x_2-x_1}$$

Mathematical modelling

ALWAYS interpret your gradient and y intercept in the context of the question!

Finding the point of intersection

Use simultaneous equations either by elimination or substitution

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Y12 – Chapter 6 Circles

What do I need to be able to do?

By the end of this chapter you should be able to:

- Find the midpoint of a line segment
- Find the equation of the perpendicular bisector to a line segment
- Know how to find the equation of a circle
- Solve geometric problems involving straight lines and circles
- Use circle properties to solve problems
- Solve problems involving circles and triangles

Key words:

- Line segment – a finite part of a straight line with two distinct end points
- Perpendicular bisector– A line which cuts a line segment into two equal parts at 90°
- Tangent – A line that just touches a curve at a point, matching the curve's slope there
- Chord –A line segment connecting two points on a curve
- Circumcircle – a circle touching all the vertices of a triangle or polygon
- Circumcentre – The center of a triangle's circumcircle

Finding midpoint of a line segment

Midpoint $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Equation of a circle

The equation of a circle with centre (a, b) and radius r is:

$$(x-a)^2 + (y-b)^2 = r^2$$

You may be given the equation of a circle in the form :

$$x^2 + y^2 - 2ax - 2by + a^2 + b^2 - r^2 = 0$$

In this case you need to complete the square for the x and y terms to find the radius and centre of the circle

Eg

$$x^2 + y^2 - 14x + 16y - 12 = 0$$
$$x^2 - 14x + y^2 + 16y - 12 = 0$$

Half the coefficient of x

Half the coefficient of y

$$(x-7)^2 - 7^2 + (y+8)^2 - 8^2 - 12 = 0$$

Subtract back off

Subtract back off

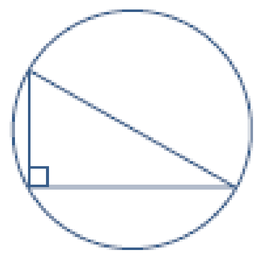
$$(x-7)^2 + (y+8)^2 = 7^2 + 8^2 + 12$$

$$(x-7)^2 + (y+8)^2 = 125$$

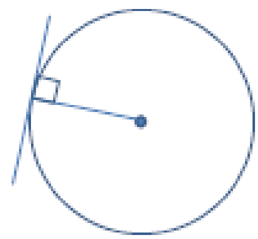
Centre (7, -8); radius $\sqrt{125} = 5\sqrt{5}$

Circle properties

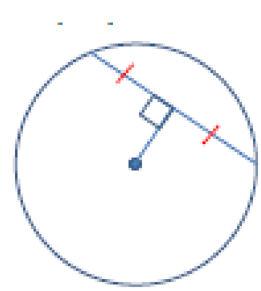
The angle in a semi circle is always a right angle



A tangent to a circle is perpendicular to the radius at the point of intersection



The perpendicular bisector of a chord will go through the centre of the circle



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Y12 – Chapter 7 Algebraic Methods

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What do I need to be able to do?

By the end of this chapter you should be able to:

- Cancel factors in algebraic fractions
- Divide a polynomial by a linear factor
- Use the factor theorem to factorise a cubic expression
- Construct mathematical proofs using algebra
- Use proof by exhaustion and disproof by counter example

Key words:

- Polynomial – A polynomial can have constants, variables (and exponents that can be combined using addition, subtraction, multiplication and division, but:
  - no division by a variable.
  - a variable's exponents can only be 0 or a positive integer.
  - not an infinite number of terms.
- Proof – Logical mathematical arguments used to show the truth of a mathematical statement.

In a proof we can use:

- axioms (self-evident truths) such as "we can join any two points with a straight-line segment" (one of Euclid's Axioms)
- existing theorems, that have themselves been proven.

Algebraic fractions

Algebraic fractions behave, and follow the same rules as numerical fractions.
When simplifying algebraic fractions, where possible factorise the numerator and denominator and then cancel out common factors

Eg Simplify

$$\frac{2x^2+11x+12}{x^2+7x+12}$$

$2x^2+11x+12$ factorises to $(2x+3)(x+4)$

$x^2+7x+12$ factorises to $(x+3)(x+4)$

So, the fraction can be written as:

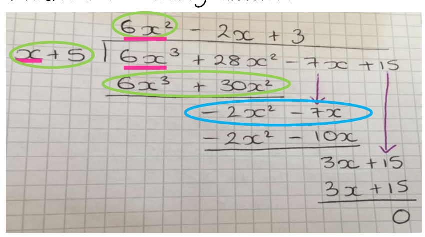
$$\frac{(2x+3)\cancel{(x+4)}}{(x+3)\cancel{(x+4)}}$$

(x+4) is the common factor so it cancels

Polynomial division

$$(6x^3+28x^2-7x+15) \div (x+5)$$

Method 1 – Long division



Divide the first term of the polynomial by x ($6x^3 \div x = 6x^2$)
Multiply (x+5) by $6x^2$ and write under polynomial
Subtract and bring down -7x

Repeat for each term of the polynomial

Method 2 – Box method

$$6x^3 + 28x^2 - 7x + 15 \quad \div \quad x+5$$

	$6x^2$	$-2x$	3
x	$6x^3$	$-2x^2$	$3x$
$+5$	$30x^2$	$-10x$	15

Divide the first term of the polynomial by x ($6x^3 \div x = 6x^2$)
Multiply +5 by $6x^2$ and write in box ($30x^2$)
Subtract $30x^2$ from x^2 term in the polynomial and complete box ($28x^2 - 30x^2 = -2x^2$ and write in box
Dived $-2x^2$ by x ($-2x$)
Multiply +5 by $-2x$ and write in box ($-10x$)
Subtract $-10x$ from the x term in the polynomial ($-7x - -10x = 3x$) and write in box
Divide 3x by x (3)
Multiply +5 by 3 and write in box (15)
If you collect the terms in your boxes it should match your polynomial

Proof

In a mathematical proof you must:

- State any information or assumptions you are using
- Show every step clearly
- Each step should follow logically from the previous step
- Make sure you have covered all possible cases
- Write a statement of proof at the end of your working

To prove an identity you should:

- Start with one side of the identity
- Manipulate it ro match the other side
- Show every step of your working

Proof continued…

Proof by exhaustion – break the statement into smaller cases and prove each one separately
Proof by counter example – give one example that does not work

Factor Theorem

If f(p) = 0, then (x – p) is a factor of f(x)
If (x – p) is a factor of f(x), then f(p) = 0

What do I need to be able to do?

By the end of this chapter you should be able to:

- Use Pascal's triangle to identify binomial coefficients and use them to expand simple binomial expressions
- Use combinations and factorial notation
- Use the binomial expansion to expand brackets
- Find individual coefficients in a binomial expansion
- Make approximations using the binomial expansion

Y12 – Chapter 8 The Binomial Expansion

Key words:

- Binomial expansion – shows us what happens when we multiply a binomial (like a+b) by itself as many times as we want.
- Binomial – A polynomial with two terms
- Factorial – to multiply all whole numbers from the chosen number down to one. The symbol is !
- Combinations – Any of the ways we can combine things, when the order does not matter

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Pascal's triangle

$$(a+b)^0 = 1$$
$$(a+b)^1 = a + b$$
$$(a+b)^2 = a^2 + 2ab + b^2$$
$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$
$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$
$$(a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$$

Use Pascal's triangle to find the coefficients

- The 1st term in the brackets starts with the power of $\boldsymbol{n}$ and decreases to 0
- The 2nd term in the brackets starts with the power of 0 and increases to $\boldsymbol{n}$

The Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n$$

$$(n \in \mathbb{N})$$

Finding Coefficients

Find the coefficient of x^4 in the binomial expansion:
$(2 + 3x)^{10}$

x^4 term $= \binom{10}{4}2^6(3x)^4$
$= 210 \times 64 \times 81x^4$
$= 1088640x^4$

So the coefficient of x^4 in the binomial expansion of $(2+3x)^{10}$ is 1088640

Approximations using the Binomial Expansion

The first four terms of the binomial expansion of $(1 - \frac{x}{4})^{10}$ in ascending order are:

$$1 - 2.5x + 2.8125x^2 - 1.875x^3$$

Use this expansion to estimate the value of 0.975^{10}

$1 - \dfrac{x}{4} = 0.975$
$x = 0.1$

$0.975^{10} \approx 1 - 2.5(0.1) + 2.8125(0.1)^2 - 1.875(0.1)^3$
$0.975^{10} \approx 0.7763\ (4sf)$

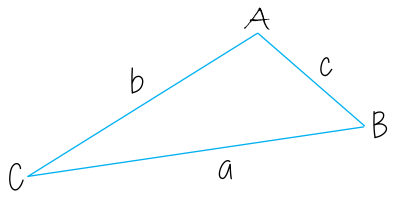
Y12 – Chapter 9 Trigonometric Ratios

What do I need to be able to do?

By the end of this chapter you should be able to:

- Use the cosine rule to find a missing side or angle
- Use the sine rule to find a missing side or angle
- Find the are of a triangle using an appropriate formula
- Solve problems involving triangles
- Sketch the graphs of the sine, cosine and tangent functions
- Sketch simple transformations of these graphs

The Cosine Rule



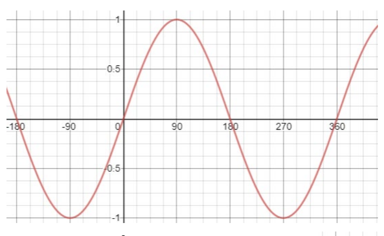
To find a missing side: $a^2 = b^2 + c^2 - 2bc\cos A$

To find a missing angle: $\cos A = \frac{b^2+c^2-a^2}{2bc}$

Use the cosine rule when you either:

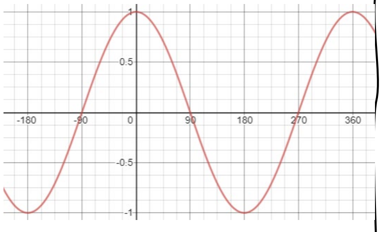
- Know two sides and the angle between them and want to know the third side
- Know three sides and want to find an angle

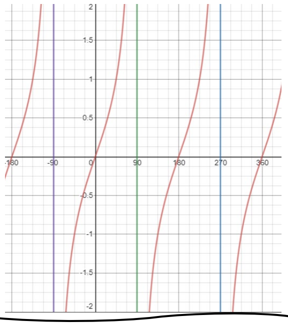
Graphs of sine, cosine and tangent



The graph of $y = \sin\theta$
Repeats every 360°
Crosses the x axis every 180°
Has a maximum value of 1 and a minimum value of -1

The graph of $y = \cos\theta$
Repeats every 360°
Crosses the x axis at -90°, 90°, 270°…
Has a maximum value of 1 and a minimum value of -1





The graph of $y = \tan\theta$
Repeats every 180°
Crosses the x axis at -180°, 0, 180°, 360°…
Has vertical asymptotes at x=-90°, x=90°, x=270°…

Key words:

- Periodic function – A function (like Sine and Cosine) that repeats forever

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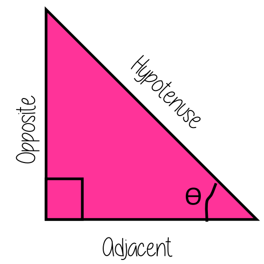
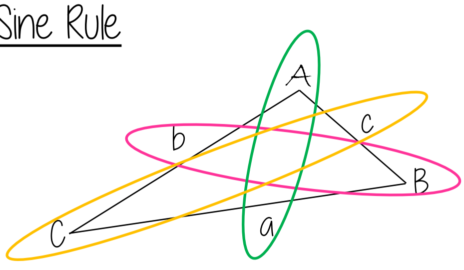
Sine, Cosine and Tangent



SINθ= Opposite/Hypotenuse
Cosθ= Adjacent/Hypotenuse
Tanθ= Opposite/Adjacent

The Sine Rule



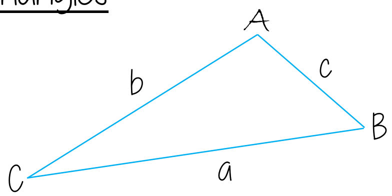
To find a missing side: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

To find a missing angle: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Use the sine rule when you have opposite pairs of angles and sides

The sine rule sometimes produces two possible solutions for a missing angle: $\sin\theta = \sin(180 - \theta)$

Areas of triangles



$$Area = \frac{1}{2}ab\sin C$$

Y12 – Chapter 10 Trigonometric identities and equations

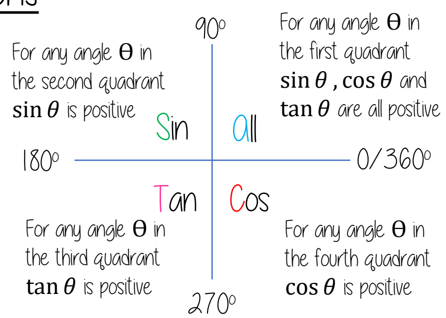
What do I need to be able to do?

By the end of this chapter you should be able to:

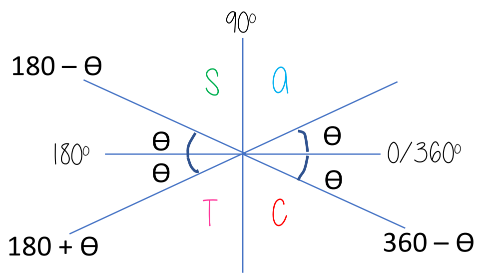
- Calculate the sine, cosine and tangent of any angle
- Know the exact trigonometric ratios for 30°, 45° and 60°
- Know and use the identities $\tan\theta \equiv \frac{\sin\theta}{\cos\theta}$ and $sin^2\theta + cos^2\theta \equiv 1$
- Solve trigonometric equations

Solving trig equations

CAST diagram



You can use the diagram to find sin, cos or tan of any positive or negative angle using the corresponding acute angle made with the x axis



$$\sin\theta = \sin(180 - \theta)$$
$$\cos\theta = \cos(360 - \theta)$$
$$\tan\theta = \tan(180 + \theta)$$
$$-\sin\theta = \sin(180 + \theta) = \sin(360 - \theta)$$
$$-\cos\theta = \cos(180 - \theta) = \cos(180 + \theta)$$
$$-\tan\theta = \tan(180 - \theta) = \tan(360 - \theta)$$

When you use the inverse trigonometric functions on a calculator, the angle you get is the principal value. Your calculator gives principal values in the ranges:

$$\sin^{-1} -90° \le \theta \le 180°$$
$$\cos^{-1}\ 0° \le \theta \le 90°$$
$$\tan^{-1} -90° \le \theta \le 90°$$

sinθ = k and cosθ= k only have solutions when $-1 \le k \le 1$
tanθ = p has solutions for all real values of p

Key words:

- Identity – An identity is an equation which is always true, no matter what values are substituted

Trig Identities

$$\tan\theta \equiv \frac{\sin\theta}{\cos\theta}$$

$$sin^2\theta + cos^2\theta \equiv 1$$

Exact trig values

	30	45	60
sin	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
tan	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

Techniques for solving trig equations

Technique	Example
Rearrange to make sin, cos or tan the subject	sin(x) – 0.3 = 0 sin(x) = 0.3 x = 17° (2sf)
Factorise if possible	3cos(x)sin(x) + sin(x) = 0 sin(x)(3cos(x) + 1) = 0 sin(x) = 0 or 3cos(x)+1=0 x = 0° or x=110° (2sf)
If it is a mixture of sin(x) and cos(x) use the identity $\tan\theta \equiv \frac{\sin\theta}{\cos\theta}$	sin(x) = 3cos(x) sin(x)/cos(x) = 3 tan(x) = 3 x = 72° (2sf)
If you have a mixture of sin and cos in a quadratic use the identity $sin^2\theta + cos^2\theta \equiv 1$ then solve	$\cos^2$(x) = sin(x) + 1 1 – $\sin^2$(x) = sin(x) + 1 $\sin^2$(x) - sin(x) = 0 Factorise and solve
Solve multiples of the unknown angle	tan(2x) = 5 2x = 79° x = 39.5°

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Y12 – Chapter 11 Vectors

What do I need to be able to do?

By the end of this chapter you should be able to:

- Use vectors in two dimensions
- Use column vectors and carry out arithmetic operations on vectors
- Calculate the magnitude and direction of a vector
- Understand and use position vectors
- Use vectors to solve geometric problems
- Understand vector magnitude and use vectors in speed and distance calculations
- Use vectors to solve problems in context

Vector forms

Form	Uses	Examples	Meaning
Component	i and j	2i – 3j	Go along 2 to the right and down 3
Column	digits	$\begin{pmatrix} -4 \\ 5 \end{pmatrix}$	Go along 4 to the left and up 5

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Magnitude of a vector

$\underline{a}$ = xi + yj = $\begin{pmatrix} x \\ y \end{pmatrix}$ magnitude of $\underline{a}$ is given by:

$$|a| = \sqrt{x^2 + y^2}$$

Position vectors

A position vector starts at the origin
eg a point A (4, -5) has position vector $\overrightarrow{OA}$ = 4i – 5j

$\overrightarrow{AO} = -\overrightarrow{OA}$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

Midpoint

$\overrightarrow{OM} = \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AB}$

Key words:

- Magnitude – The magnitude of a vector is its length (ignoring direction)
- Resultant – the vector sum of two or more vectors
- Scalars – A single number (used when dealing with vectors or matrices)

Adding and multiplying vectors

$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

To add vectors algebraically you add the i and j components eg:

$\underline{a}$ = 3i + 5j $\underline{b}$ = i – 7j

$\underline{a} + \underline{b}$ = 4i – 2j

To multiply by a scalar, multiply each component eg:
$\underline{a}$ = 3i + 5j

4$\underline{a}$ = 4(3i + 5j) = 12i + 20j

$\underline{b} = \begin{pmatrix} -2 \\ 6 \end{pmatrix}$
$3\underline{b} = 3\begin{pmatrix} -2 \\ 6 \end{pmatrix} = \begin{pmatrix} -6 \\ 18 \end{pmatrix}$

Unit vectors

A unit vector in the direction of $\underline{a}$ is $\frac{a}{|a|}$

Parallel vectors

Any vector parallel to $\underline{a}$ can be written as $\lambda\underline{a}$ where λ is a non-zero scalar

If $\underline{a}$ and $\underline{b}$ are two non parallel vectors and $p\underline{a} + q\underline{b} = r\underline{a} + s\underline{b}$ then p=r and q=s

Y12 – Chapter 12 Differentiation

What do I need to be able to do?

By the end of this chapter you should be able to:

- Find the derivative of a simple function
- Use the derivative to solve problems involving gradients, tangents and normal
- Identify increasing and decreasing functions
- Find the second order derivative
- Find stationary points of functions and determine their nature
- Sketch the gradient function of a given function
- Model real life situations with differentiation

Differentiating from first principles

It's a proof so you have to show ALL steps use the formula, substituting in the function

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Differentiating

If $y = ax^n$ then $\frac{dy}{dx} = anx^{n-1}$

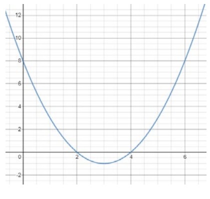
If $f(x) = ax^n$ then $f'(x) = anx^{n-1}$

When differentiating you multiply each term by its power and then reduce its power by 1

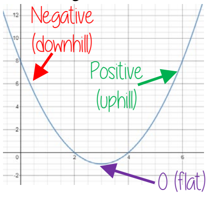
Sketching gradient functions

To sketch the gradient function, think about what is happening to the gradient at various points on the curve and sketch them

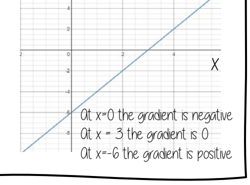
Curve



Thinking about the gradients



Gradient function



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Key words:

- Derivative – a way to show rate of change: that is, the amount by which a function is changing at one given point
- Stationary point – A point on a curve where the slope is zero. This can be where the curve reaches a minimum or maximum

Notation and definitions

The gradient of a curve at a given point is defined as the gradient to the tangent to the curve at that point

The gradient function or derivative of the curve $y = f(x)$ is written as $f'(x)$ or $\frac{dy}{dx}$ or y' or $\frac{\delta y}{\delta x}$

The gradient function ($\frac{dy}{dx}$) measures the rate of change of y with respect to x

Tangents and normals

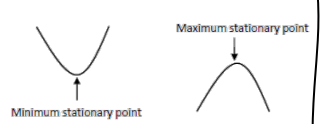
The tangent to the curve $y = f(x)$ at the point $(a, f(a))$ has the equation:

$y - f(a) = f'(a)(x - a)$

The normal to the curve $y = f(x)$ at the point $(a, f(a))$ has the equation:

$y - f(a) = 1/f'(a)(x - a)$

Stationary points



Solving $\frac{dy}{dx} = 0$ gives the x coordinate of the stationary points. Sub x value into $y = f(x)$ to find the y coordinates

Solving $\frac{d^2y}{dx^2} = 0$ gives the nature of the stationary point. If $\frac{d^2y}{dx^2} > 0$ then it's a minimum. If $\frac{d^2y}{dx^2} < 0$ then it's a maximum

Y12 – Chapter 13 Integration

What do I need to be able to do?

By the end of this chapter you should be able to:

- Find y given $\frac{dy}{dx}$ for x^n
- Integrate polynomials
- Find f(x) given f'(x) on a point on the curve
- Evaluate a definite integral
- Find the area bounded by a curve and the x-axis
- Find areas bounded by curves and straight lines

Indefinite integration

$$\int ax^n dx = \frac{ax^{n+1}}{n+1} + c \quad n \neq 1$$

This expression is the integrand

If you are integrating a polynomial function, you integrate each term one at a term

To find the constant of integration, c:

1) Integrate the function
2) Substitute the coordinates of a point on the curve into the integrated function
3) Solve the equation to find c

Definite integration

A definite integral has limits. To evaluate a definite integral you integrate as normal and then substitute the top limit and the bottom limit and subtract

Upper limit

$$\int_b^a gx^n dx = \left[\frac{gx^{n+1}}{n+1}\right]_b^a = \left(\frac{ga^{n+1}}{n+1}\right) - \left(\frac{gb^{n+1}}{n+1}\right) n \neq 1$$

Lower limit

You don't need the +C with definite integration as you are going to subtract so it cancels out

Definite integrals give you the area under the curve between the limits

Key words:

- Integral – the result of integration
- Integrand – The function we want to integrate

Notation and definitions

Integration is the reverse of differentiation

$$\int (3x^5 + 7x^2 - 4x + 2)dx$$

Means integrate the following — With respect to x

Areas under curves

The area between a positive curve, the x-axis and the lines x=a and x=b is given by

$$Area = \int_b^a y\, dx$$

Where y = f(x) is the equation of the curve

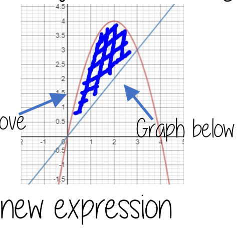
A positive answer means that the area is above the x-axis

A negative answer means that the area is below the x-axis

If there is a mixture of areas above and below the x-axis you have to work out each area separately and add them together (ignoring the negative sign)

To find the area between a curve and a line:

1) Find the x coordinate of the points of intersection
2) Subtract the equation of the graph that is below from the equation of the graph that is above



3) Integrate your new expression
4) Substitute in your x coordinates as limits to find the area

Y12 – Chapter 14 Exponentials and Logarithms

What do I need to be able to do?

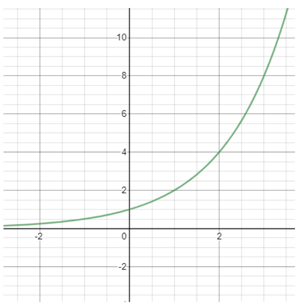
By the end of this chapter you should be able to:

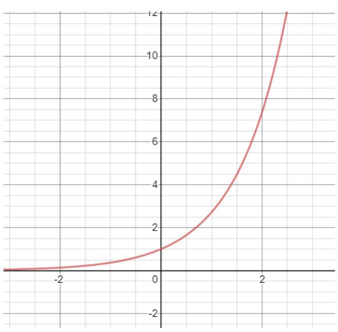
- Sketch graphs of the form $y=a^x$, $y=e^x$, and transformations of these graphs
- Differentiate e^{kx}
- Use and interpret models that use exponential functions
- Recognise the relationship between exponential and logarithms
- Recall and apply the laws of logarithms
- Solve equations in the form $a^x = b$
- Describe and use natural logarithms
- Use logarithms to estimate the values of constants in non-linear models

Exponential functions

$y = a^x$

Always crosses the y-axis at 1
The x-axis is an asymptote





$y = e^x$

The number "e" is one of the most important numbers in mathematics. The first few digits are:

2.718281828459045235360287471352 7

If $y = e^{kx}$ then $\frac{dy}{dx} = ke^{kx}$

Logarithmic graphs

For equations in the form $y = kx^n$ or $y = ab^x$ we can take logs to transform the curves into straight lines

Original	$y = kx^n$	$y = ab^x$
Take logs of both sides	$\log(y) = \log(kx^n)$	$\log(y) = \log(ab^x)$
Use laws of logs to get in the form $y = mx + c$	$\log(y) = n\log(x) + \log(k)$	$\log(y) = x\log(b) + \log(a)$
Gradient	n	$\log(b)$
Y-intercept	$\log(k)$	$\log(a)$

Key words:

- Exponential – a function in the form $f(x) = ab^x$
- Logarithm – A logarithm answers the question "How many of this number do we multiply to get that number?

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Logarithms

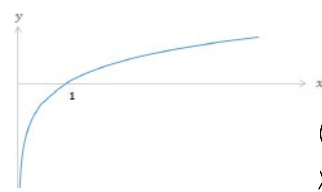
$\log_a n = x$ is equivalent to $a^x = n$

$$a \neq 1$$

Natural logarithms

Natural logarithms are logs in the base of e. Ln and e are the inverse of each other so they will cancel each other out

$$\ln e^x = x$$
$$e^{\ln x} = x$$



$y = \ln x$

Always crosses the x-axis at 1
The y-axis is an asymptote

Laws of logarithms

$$\log x + \log y = \log(xy)$$
$$\log x - \log y = \log\left(\frac{x}{y}\right)$$
$$\log(x^k) = k\log x$$
$$\log 0 = 1$$

Solving equations is the form $a^x = b$

1) Take logs of both sides
2) Use the power law to bring the power to the front
3) Solve the equation as normal